M.Sc. - I (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH01: - Group Theory & Ring Theory

P. Pages : 2 **GUG/S/25/13737**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each answer carries equal marks.

UNIT-I

1. a) Prove that, every group is isomorphic to a permutation group. **10**

b) Prove that, if N and M are normal subgroup of G such that $N \cap M = \{e\}$ then $nm = mn, \ \forall\, n \in N, m \in M$. **10**

OR

c) Let H and K be normal subgroups of G and $K \subset H$. Then prove that $(G/K)/(H/K) \simeq G/H$. **10**

d) Prove that, every group of order P^2 (P is prime) is abelian. **10**

UNIT-II

2. a) Prove that, any two composition series of a finite group are equivalent. **10**

b) Let G be a nilpotent group then prove that every subgroup of G and every homomorphic image of G are nilpotent. **10**

OR

c) Let A_n is a normal subgroup of S_n. Prove that $A_n (n > 1)$ is of index 2 in S_n and $|A_n| = n!/2$. **10**

d) Prove that, every permutation can be expressed as a product of transposition. **10**

UNIT-III

3. a) Let G be a finite group and let P be a prime then prove that if p^m divides $|G|$ then G has a subgroup of order p^m. **10**

b) Let G be a group of order pq, where p and q are prime numbers such that $p > q$ and $q/(p-1)$. then prove that G is cyclic. **10**

OR

c) Prove that, if a group of order P^n contains exactly one subgroup each of orders $P^1, P^2, ...P^{n-1}$. Then it is cyclic. **10**

d) Prove that there are no simple groups of order 63, 56 and 36. **10**

UNIT-IV

4. a) Let $f : R \to S$ be a monomorphism of a ring R into a ring S. Then prove that $\ker f = \{0\}$ if and only if f is $1-1$. **10**

b) Prove that, if a ring R has unity then every ideal I in the matrix ring R_n is of the form A_n, where A is some ideal in R. **10**

OR

c) Prove that, in a nonzero commutative ring with unity, an ideal M is maximal if and only if R/M is a field. **10**

d) Prove that, if R is a commutative Ring then an ideal P in R is prime if and only if $ab \in P$, $a \in R, b \in R$ then $a \in P$ are $b \in P$. **10**

5. a) Let G be a group and G' be the derived group of G then prove that $G' \Delta G$. **5**

b) Let ϕ, ψ be mapping from n to n then prove that $E(\phi \cdot \psi) = E(\phi) \cdot E(\psi)$. **5**

c) Prove that, a sylow P subgroup of a finite group G is unique if and only if it is normal. **5**

d) Prove that, if D is division ring then $R = D_n$ has no nontrivial ideals. **5**

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